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SIGNIFICANCE OF DETONATION STUDY TO PROPULSION DYNAMICS*

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Abstract

The various means by which detonation study can influence the development of propulsion systems are discussed. The wave, besides the possibility of its actual appearance in the course of anomalous operation of the combustion chamber, or its direct exploitation as the actual combustion front in an engine specially designed for this purpose, provides the prime experimental facility for the study of non-steady gasdynamics of systems chracterized by a high rate of energy release. Moreover it offers an insight into the relationship between the chemico-kinetic and gasdynamic phenomena that occur in thrust chambers not only during transient processes but also at steady state operation. Its theoretical aspects provide a stimulus to the development of gas-wave-dynamics, the subject dealing with the analysis of wave interaction processes, whose techniques are applicable not only to chemical propulsion systems, but also to gaseous nuclear devices and pulsed plasma accelerators.

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1. Introduction

The progress of propulsion has been marked by improvements in the level of power density of heat release in the gaseous medium that is used as the working substance. Fig. 1 represents the spectrum of such power density levels over a full range of prime-mover systems used for propulsion, from steam engine to rocket boosters. The scope of the detonation wave in this respect is included for comparison. Its relative compactness is due, of course, to its small thickness, but this should not detract from the utility of the wave as a convenient laboratory tool for the study of the processes of comparably intense heat release that occurs in the thrust chambers of booster rockets. The great versatility of the detonation wave in this connection is promoted by the fact that, since the heat release per unit volume is proportional to density while the thickness of the wave is inversely proportional to pressure, power density level is proportional to square of the initial pressure. It can be therefore easily adjusted to fit the particular conditions prevailing in the thrust chamber whose performance one wishes to study.

Another important class of problems where the detonation wave can be used to great advantage concerns combustion in two-phase mixtures. By observing the development of detonation through a cloud of fuel droplets in an oxidizer gas one can learn for instance a good deal about spray combustion, let alone its ability to generate pressure waves.

The most general gain however that could result from detonation study may be derived from the incentive it provides to the development of the

subject of gas-wave-dynamics - the physico-chemistry of processes that occur at wave fronts and the dynamics of wave interaction phenomena. In this respect the study transcends the combustion reaction and lends itself to research concerned with advanced propulsion systems, notably the dynamics of fissionable gases that is of direct interest to nuclear propulsion, and the analysis of wave interaction processes which take place in pulsed plasma accelerators.

The importance of detonation research to rocket technology stems therefore primarily from the fact that, besides the possibility of its direct exploitation, the detonation wave produces conditions resembling quite closely those prevailing in high performance thrust chambers so that it can serve as convenient means for the study of their chemico-kinetic and gas-wave-dynamic processes. Of a more particular interest is the relationship between the kinetics of the combustion reaction and the dynamics of pressure waves whose formation constitutes a prominent feature of the process of heat release at a high power density level. The performance of such high power density systems depends crucially upon the success in preventing their unstable operation. The instabilities are, as a rule, manifested by the appearance of intense and destructive pressure waves. Knowledge of the mechanism by which such waves are formed represents therefore an essential requirement for a successful development of high performance propulsion systems.

Recent progress of detonation study has been summarized in a publication [1] presented at the last Summer Meeting of the AIAA in Los

Angeles, while some of its gas-wave-dynamic aspects, that bear most directly on the subject matter of the present communication, have been described in a paper [2] currently under preparation for presentation at the VI Symposium on Advanced Problems in Fluid Mechanics in Poland. Discussed here are the various means that have been proposed for direct exploitation of the detonation wave for rocket propulsion; the significance of detonation study to the investigation of transient processes, distributed combustion, and reactions in two-phase mixtures under conditions of power density level and temperature corresponding to those prevailing in thrust chambers of booster engines; and finally the application of the knowledge acquired as a result of detonation studies to the analysis of advanced propulsion systems, namely the gaseous fission reactor and the pulsed plasma accelerator.

2. Detonation Wave Engines

The most obvious significance of detonation study to propulsion is brought out by schemes concerned with direct utilization of the wave in the thrust chamber of an engine. Three modes of operation have been considered in this respect: the stationary mode, particularly appropriate for ramjet application; the intermittent mode for possible use in a pulsejet and the rotating mode, especially suited for a rocket thrust chamber.

The stationary detonation wave engine was studied by Dunlap, Brehm, and Nicholls [3], Weber and MacKay [4], and Sargent and Gross [5]. All of them considered designs based primarily on the use of a Chapman-

Jouguet detonation, although the latter have admitted the possibility of strong detonations. With the use of appropriate diffuser geometry, Fig. 2, the Mach number of C-J detonations in hydrocarbon-air mixtures could be varied from approximately 2.5 to 4.5, giving an acceptable performance over a range of flight Mach numbers from 3 to 10. Although the optimum performance of a detonation engine is always below that of a conventional ram-jet, its main advantages lie in the fact that it has a much narrower combustion zone and gives good promise for stable operation (Nicholls, et al [6, 7, 8], Gross and Chinitz [9], Rhodes, et al [10]).

The intermittent engine has been investigated both experimentally and theoretically by Nicholls, Wilkinson and Morrison [11]. Their experimental apparatus consisted of a 1 inch dia. x 6 ft. long pipe, provided with a continuously operating induction and mixing system for fuel and air, Fig. 3. It was operated at frequencies from 10 to 35 cps, using hydrogen-air mixtures, and produced an encouraging agreement between experimental and theoretical results.

Work on development of a rotating detonation wave motor is currently conducted by J. A. Nicholls and R. E. Cullen at the University of Michigan [12]. In their device, Fig. 4, a rotating combustion wave of detonative character is setup in the annular chamber by means of a suitable ignition procedure. The wave continues then to propagate azimuthally, while the main flow of reactants and products is in the axial direction. Interestingly enough, a similar system has been employed quite independently in Novosibirsk by Voitsekhosky [13, 14] in order to study the properties of a steady detonation wave.

3. Detonation and Transient Processes

The most widespread opinion about the significance of detonation studies to rocket combustion stems from the belief that it actually occurs in the course of unstable operation. The question whether existence of a detonation wave is or is not associated with combustion instability is quite academic and essentially beside the point. The fact, however, that the instability is manifested by the appearance of intense pressure waves whose interaction with the combustion reaction may become a dominating factor is unquestionable. It is with this in mind that the subject matter of detonation study can be looked upon as one concerned in a more general sense with the fundamental aspects of energy release in the presence of finite pressure gradients. In this connection studies of pre-detonative phenomena should be particularly significant.

The insight that one may gain into the mechanism of the generation of pressure waves by a combustion process may be illustrated by the following experimental results obtained recently in our laboratory [15, 16, 17, 18]. They are all concerned with the development of detonation in stoichiometric hydrogen-oxygen mixtures contained initially at NTP in a 1" x 1-1/2" cross-section detonation tube fitted with windows across the whole 1.0 inch width of the test section.

Fig. 5 demonstrates the early stages of the process initiated by a pilot flame which has been admitted to the observation section through a 1/32" dia. x 1/2" long orifice from a pre-combustion chamber of 10 cm^3 volume. The streak Schlieren photograph shows the trace of the accelerating

flame, with a system of coalescing pressure waves ahead. Included as inserts are pressure records obtained at the three positions marked on the photograph. The vertical grid is 10.4 psi/div for insert 1 and 26 psi/div for inserts 2 and 3. The horizontal grid is 50 μsec/div. Fig. 6 shows the corresponding sequence of flash Schlieren photographs taken across the full width of the detonation tube. An interpretation of the latter is depicted in Fig. 7.

As it appears from these records, the pilot flame enters the test chamber in the form of a supersonic jet associated with a bow shock wave which soon detaches from the flame carrying behind a train of shock fronts. Thereupon the combustion front propagates as a wrinkled laminar flame, while the system of pressure waves collapse to form an accelerating head shock and a receding rarefaction. Close to the upper edge of the streak Schlieren record and, as evident in the last flash photograph, the flame eventually breaks up into a turbulent, tulip-shaped brush. The reaction zone becomes then distributed and the process gains added momentum for its acceleration.

The most significant contribution gained from such observations is in the understanding that they provide concerning the mode of the combustion process at the most crucial time from the point of view of rocket combustion, that is, when it acts as the source of pressure waves. The advances in the gasdynamic theory of the pressure waves generated by accelerating flames have been reported by us in the Proceedings of the Royal Society [15] and the Ninth International Symposium on Combustion [18]. Their synthesis is presented also in a more recent publication [2].

As the process develops further, the turbulent combustion zone spreads out more and more, while the system of pressure waves ahead of it becomes reinforced until, as shown in Fig. 8, there occurs "an explosion in explosion". This gives rise to a retonation wave - a shock wave propagating into the burned gases - and a superdetonation front - a shock front propagating into the unburned mixture. At the same time there is generated a system of transverse waves which are recorded on the streak photograph as a family of hyperbola-like traces. The origin of the transverse waves and their exact nature is shown in two flash Schlieren photographs included as inserts in Fig. 8. They are located at the time instances to which they correspond, each however having an arbitrary size of the space coordinate. As indicated by the flash photographs, the combustion zone is at that time quite widely distributed and the transverse waves originate evidently from a point explosion that occurs near the wall. Similar observations have been made recently by Soloukin [19] who obtained his records from a streak Schlieren system oriented along the axis of the tube with the process viewed from the end across the width of the test section.

Although the phenomenon of the "explosion in explosion" may provide the key to the understanding of the onset of high amplitude pressure waves in a variety of combustion systems, ranging from knock in I.C. engines to high frequency combustion instability in rocket thrust chambers, neither its exact nature nor cause has been as yet adequately explored.

As contrasted to the insight into the mechanism of unstable combustion

that is gained by detonation studies, most of the work which has been done so far on combustion instabilities in thrust chambers could be looked upon as that concerned with the servo-mechanism or system dynamics of the phenomenon.

The literature on combustion instability is today so extensive that its mention has to be considered outside the scope of this paper. Among the many treatments of the problem there is only one, known to us, where the similarity between detonation and combustion in the thrust chamber has been actually exploited in order to derive a stability criterion. It has been first proposed by Shchelkin, and then presented in a joint paper with Denisov and Troshin [20] .

The derivation of the criterion is based on the concept that the combustion zone in a thrust chamber can be treated as a one-dimensional deflagration, similarly as it is done in the classical, Von Neuman-Döring-Zeldovich theory of detonation. According to this model, the stability is ascertained if a temperature disturbance, δT, downstream cannot reach the upstream boundary of the combustion zone, that is if

$$\left| \delta T \, \overline{\frac{dt}{dT}} \right| < \tau \tag{3.1}$$

where τ is the time lag between the upstream and downstream boundary of the combustion zone and $\overline{\frac{dT}{dt}}$ is the average temperature gradient, with respect to time, of particle transit across this zone. Since under the assumption of the classical theory the extent of the combustion zone is governed by kinetics, the transit time is directly related to temperature by

some overall Arrhenius expression, so that:

$$t = k \exp(E^{\ddagger}/RT) \tag{3.2}$$

where k and $E^{\ddagger}/R$ are considered for the present purpose to be constant. The time lag, τ, is evaluated with $T = T_x$, subscript x denoting the upstream conditions while y is used to denote the conditions at the downstream boundary of the combustion zone. Assuming a polytropic relation between pressure and density, Eqs. (3.1) and (3.2) yield the following "Shchelkin stability criterion" applicable to any deflagrative process:

$$\frac{E^{\ddagger}/R}{T_x}\left[1-\left(\frac{p_y}{p_x}\right)^{\frac{\gamma-1}{\gamma}}\right] < 1 \tag{3.3}$$

In a thrust chamber $p_x = p_y - \delta p$ where δp is a small pressure perturbation. Hence (3.3) reduces to:

$$\frac{\gamma-1}{\gamma}\;\frac{E^{\ddagger}/R}{T_x}\;\frac{\delta p}{p_x} < 1 \tag{3.4}$$

The pressure change, δp, can be related to the mass flow rate per unit area, $\dot{m}$, and the heat release per unit mass, q, by the following approximate formula:

$$\delta p = p_x - p_y \approx \frac{\gamma-1}{\gamma}\,\frac{\dot{m}q}{p} \tag{3.5}$$

which expresses the pressure drop along a Rayleigh line between the initial state, x, and its intersection, y, with a Hugoniot curve corresponding to q and plotted with reference to state x. With (3.5), Eq. (3.4) yields

finally the following stability criterion:

$$\left(\frac{\gamma-1}{\gamma}\right)^2 \frac{\dot{m}^2 (E^{\ddagger}/R)\, q}{p_x^2 \, T_x} < 1 \qquad (3.6)$$

For a representative case of $\gamma = 1.25$, $\dot{m} = 1000$ lbs/ft^2 sec, $E^{\ddagger}/R = 20{,}000^\circ$R, $q = 4000$ BTU/lb, $p_x = 500$ psi and $T_x = 1000^\circ$R, the criterion attains a value of 15, indicating indeed a condition of intrinsic instability.

Of course, a possibility of existence is by no means a sufficient condition for the occurrence of unstable operation. A complete instability analysis should consider not only the sustenance of the deviations from equilibrium, usually manifested by finite amplitude oscillations, but should inquire also into the driving mechanism.

Most of the theories on combustion instability in rocket thrust chambers are associated with some aspects of the former and, in fact, are particularly concerned with the specification of conditions under which the deviations can be sustained. The "Shchelkin criterion" is a representative example of such an approach.

The latter has received so far very little attention. It is however of interest to note that recent studies of combustion instability in liquid propellant rocket motors (Sirignano [21]) lead to the conclusion that the axial mode of oscillations in a combustion chamber can have a periodic character only if it is associated with the action of a shock front. The analysis is derived from the theory of thermally driven non-linear oscillations in a pipe that has been developed by Boa-Teh Chu [22] whose

approach has been also followed by us in order to interpret the experimentally observed gereration of pressure waves by accelerating flames in explosive mixtures.

4. Detonation and Distributed Combustion

With the progress of our knowledge about the combustion process in a thrust chamber, it becomes more and more obvious that its consideration as a discrete reaction front is an over-simplification. It is indeed remarkable that the understanding of the detonation wave, gained as a result of recent investigations, leads to exactly the same conclusion. Fig. 9 presents the experimental observations that demonstrate this point. On the left is the imprint left by a detonation wave on the soot-coated wall of a tube through which it has travelled. The record has been obtained by R. E. Duff [23], but the technique was used originally by Denisov and Troshin [24]. It reveals the interaction pattern around the full periphery of a 5/8" dia. tube produced by a detonation wave in a stoichiometric hydrogen-oxygen mixture, initially at pressure of an order of 10 cm Hg. The record indicates the inherent instability of a detonation wave. In contrast to the classical concept of a plane wave, the interaction pattern suggests that the wave is made up of a number of heads, each presumably representing the apex of a triple shock interaction, which propagate in a direction transverse to the main motion.

The middle record is an interferogram obtained by D. R. White [25] of a self-sustained detonation in a $2H_2 + O_2 + 2CO$ mixture that has been

initially maintained at a pressure of 0.3 atm and room temperature in an 8.25 cm square inch shock tube. The apparently turbulent character of the combustion zone can be interpreted as another manifestation of the same phenomena as those that produced the complex imprint on the soot-coated wall.

In his recent publication White, with K. H. Cary, [26] demonstrated how such instability can be suppressed, permitting the attainment of a laminar wave, as that demonstrated on the right side of Fig. 9. The record has been obtained by propagating a self-sustained detonation through a nozzle formed by inserting a wedge across the full width of the tube. The interferogram shows the wave as it emerges from the divergent portion of the nozzle, whose wall is just evident in the upper left hand corner of the record. An induction zone exists immediately behind the smooth shock front, separating it from the laminar reaction zone. Measurement of the thickness of the induction zone and the corresponding density can be then used for the evaluation of the overall rate of the reaction process, producing thus a link between the gasdynamic aspects of wave processes and the kinetics of the reaction that constitutes the basic driving mechanism.

The most significant feature of the experimental observations exemplified by Fig. 9 is that they demonstrate the distributed character of the combustion process in the detonation wave and provide an insight into its structure. Moreover they also show how the intrinsic instabilities of the process can be gasdynamically controlled.

5. Detonation in Two-Phase Mixtures

Of particular importance to thrust chamber technology is spray combustion, and the detonation wave has attracted some attention in this respect as well. Williams [27, 28] studied the parameters and structure of the steady wave analytically with the view of its direct exploitation in a liquid rocket system. Webber [29] and Cramer [30] reported on experiments that have been performed as part of the investigation of combustion instabilities in thrust chambers. Because of its significance to the subject matter of our paper, Williams' analysis is here reconstructed from basic principles.

The properties of the steady state spray detonation can be found from the ordinary fluid dynamic equations which have been suitably modified to account for the average effect of the droplets. For this purpose consider a heterogeneous mixture comprised of N components of gaseous species of kind k and M components of liquid droplets of kind j. Assuming that there are no sources or sinks and no droplet collisions, and that all droplets disappear downstream of the wave, the equations of conservation of mass, momentum, and energy and the equation of state can be written as follows:

$$\rho_{f\infty} u_\infty = \rho_{fo} u_o + \dot{m}_s \tag{5.1}$$

$$\rho_{f\infty} u_\infty^2 + p_\infty = \rho_{fo} u_o^2 + p_o + P_s \tag{5.2}$$

$$\rho_{f\infty} u_\infty \left(h_{f\infty} + \frac{u_\infty^2}{2} \right) = \rho_{fo} u_o \left(h_{fo} + \frac{u_o^2}{2} \right) + E_s \tag{5.3}$$

$$p_\infty / \rho_{f\infty} R_{f\infty} T_{f\infty} = p_o / \rho_{fo} R_{fo} T_{fo} \tag{5.4}$$

where ρ_f is the gas density, u - the gas velocity, T_f - the gas temperature, p - the pressure, h_f - the enthalpy per unit mass of the gas, R - the specific gas constant, subscripts o and ∞ denote conditions ahead and behind the wave respectively and the terms $\dot{m}_s$, P_s, and E_s represent the droplet contribution to the flux of mass, momentum and energy respectively.

Since it is physically reasonable to assume that initially all droplets have essentially the same velocity as the gas, it becomes convenient to define the mass flux fraction of the spray as $Z \equiv \rho_s / \rho_o$, where ρ_o is the total density of the initial mixture and the spray density, ρ_s, is given by $\dot{m}_s / u_o$. Both Z_o and ρ_s, of course, represent the total contribution from the M liquid species, i.e. $Z_o \equiv \sum_{j=1}^{M} Z_{jo}$ and $\rho_s = \sum_{j=1}^{M} \rho_{sj}$. In this instance then:

$$\dot{m}_s = u_o \sum_{j=1}^{M} \rho_{sj}$$

so that

$$P_s = \dot{m}_s u_o$$

and

$$E_s = \sum_{j=1}^{k} \rho_{sj} u_o \left(h_{l,jo} + \frac{u_o^2}{2} \right)$$

where $h_{\ell,jo}$, the enthalpy per unit mass of liquid droplet species j is assumed independent of the droplet size. With these expressions Eqs. (5.1) to (5.4) become:

$$\rho_\infty u_\infty = \rho_o u_o \tag{5.5}$$

$$\rho_\infty u_\infty^2 + p_\infty = \rho_o u_o^2 + p_o \tag{5.6}$$

$$h_{f\infty} + \frac{u_\infty^2}{2} = (1-Z_o) h_{fo} + \frac{u_o^2}{2} + \sum_{j=1}^{M} Z_{jo} h_{\ell,jo} \tag{5.7}$$

$$p_\infty/\rho_\infty R_\infty T_\infty = p_o/\rho_o R_{fo}(1-Z_o) T_{fo} \tag{5.8}$$

For comparison with purely gaseous detonations, consider the above set of equations reduced to the special case of constant molecular weights ($R_\infty = R_o$), and specific heats, c_p, of all gaseous species. Expressing then the enthalpy of gaseous species k as follows:

$$h_{fk} = [h_k^o + c_p(T_f - T^o)] Y_k \tag{5.9}$$

one obtains from (5.7) and (5.8):

$$c_p T_{f\infty} + \frac{u_\infty^2}{2} = c_p(1-Z_o) T_{fo} + \frac{u_o^2}{2} + Q \tag{5.10}$$

$$\frac{p_\infty}{\rho_\infty T_\infty} = \frac{p_o}{\rho_o(1-Z_o)T_{fo}} \tag{5.11}$$

where

$$Q = \sum_{J=1}^{M} Z_{Jo} h_{\ell,Jo} + \sum_{k=1}^{N} \left[h_k^o + c_p(T_{fo} - T^o)\right]\left[Y_{ko}(1-Z_o) - Y_{k\infty}\right] + Z_o c_p T_{fo}$$

h_k^o being the standard enthalpy of formation per unit mass of gaseous species k at standard reference temperature T^o and Y_k the mass fraction of gaseous species k. With the exception of the equation of state and the form of the term representing the effective heat of reaction, the resulting Eqs.(5.5), (5.6), (5.9) and (5.11)for the heterogeneous system, which are the same as those deduced by Williams [27], are formally equivalent to those for a purely gaseous medium. If the definitions of the initial temperature and the heat of reaction are suitably modified for the spray, the two sets of equations become identical while, for the same initial conditions and heat release, Williams found only minor differences between the Chapman-Jouguet states, the most significant being a 10% increase in the pressure ratio for the case of detonation in a dilute spray.

To analyze the structure of spray detonation we impose the further restriction, valid for small Weber number, that the spray is comprised only of spherical droplets which are of the same radius, r, and travel with the same velocity, V. The relations expressing, respectively, the

rate of growth, $\dot{r}$, and the acceleration, F, of the droplets can be then written as:

$$v\,dr/dx = \dot{r} \tag{5.12}$$

$$v\,dv/dx = F \tag{5.13}$$

where $\dot{r}$ and F are identical for all species j.

Since the velocity of the droplets differs, in general, from that of the gas, the mass flux fraction of the spray can be no longer expressed as the ratio of spray density to the density of the mixture, but now must be defined as:

$$Z \equiv \frac{\rho_s v}{\dot{m}} = \frac{\sum_{J=1}^{M} \rho_{sj} v}{\dot{m}} = \sum_{J=1}^{M} Z_j \tag{5.14}$$

where $\dot{m}$ is the total flow rate per unit area for droplets and gas. Introducing n_j to denote the number of droplets of kind j per unit volume at position x, and $\rho_{\ell j}$ as the density of droplets of kind j then Eq. (5.14) becomes:

$$Z = \frac{\sum_{J=1}^{M} \frac{4}{3}\pi r^3 \rho_{\ell j} n_j v}{\dot{m}}$$

With this expression, Eq. (5.12) determines then the gradient of Z:

$$\frac{dZ}{dx} = \left(3Z_o^{1/3}/r_o\right) Z^{2/3}\, \dot{r}/v \tag{5.15}$$

To estimate the maximum possible deviation of a spray detonation from a purely gaseous wave, one can restrict the argument to the limiting case of negligible homogeneous reactions in comparison to the heterogenoues

processes. In this instance the conservation equations and the equation of state for the gas become:

$$\rho_f u + \rho_s v = \dot{m} \tag{5.16}$$

$$\rho_f u^2 + \rho_s v^2 + p - \mu du/dx = P \tag{5.17}$$

$$\rho_f u(h_f + \tfrac{1}{2}u^2) + \sum_{j=1}^{M} \rho_{sj} v(h_{\lambda,jo} + \tfrac{1}{2}v^2) - \lambda \frac{dT_f}{dx} - \mu u \frac{du}{dx} = E \tag{5.18}$$

$$p = \dot{m} R T_f (1-z)/u \tag{5.19}$$

where $\dot{m}$, P, and E are constants, μ is the coefficient of viscosity of the gas, λ - the thermal conductivity of the gas, and the remaining symbols have the same meaning as before. The gas density in Eq. (5.19) has been replaced by the expression

$$\rho_f = \dot{m}(1-z)/u \tag{5.20}$$

which follows directly from the definition of z, Eq. (5.14). Combining Eqs. (5.14), (5.17), (5.19) and (5.20) we obtain

$$du/dx = -(\dot{m}/u)\left[(P/\dot{m}) - (1-z)u - zv - RT_f(1-z)/u\right] \tag{5.21}$$

The expression of the conservation of mass for chemical species k in the gas is:

$$\frac{d}{dx}(\rho_f u Y_k) = -\Sigma \frac{d}{dx}(\epsilon_{k,j} \rho_{s,j} v) \tag{5.22}$$

where $\epsilon_{k,j}$ is the mass of species k added to the gas per unit mass of liquid vaporized from a droplet of kind j. Assuming $\epsilon_{k,j}$ constant, and using Eqs. (5.14) and (5.20), Eq. (5.22) can be integrated immediately to yield:

$$Y_k = \left[Y_{ko}(1-Z_o) - \left(\sum_{j=1}^{M} \frac{\epsilon_{kj} Z_{jo}}{Z_o} \right)(Z - Z_o) \right] \Big/ (1-Z) \tag{5.23}$$

which, far downstream of the wave, reduces to:

$$Y_{k\infty} = Y_{ko}(1-Z_o) + \sum_{j=1}^{M} \epsilon_{k,j} Z_{jo} \tag{5.24}$$

Assuming that h_{fk} is expressed, as previously, by Eq. (5.9) and that $h_{\ell,jo}$ is in addition independent of temperature, then, with the use of Eqs. (5.14), (5.20), (5.23) and (5.24), Eq. (5.18) can be written in the following form.

$$\dot{m}\left\{(1-Z)\left[c_p(T_f - T_{fo}) + \frac{u^2}{2}\right] + Z\frac{v^2}{2} + \frac{Z}{Z_o}\hat{Q}\right\} - \lambda\frac{dT_f}{dx} - \mu u\frac{du}{dx} = \hat{E} \tag{5.25}$$

where

$$\hat{Q} \equiv \sum_{J=1}^{M} z_{jo} h_{\ell,jo} + \sum_{k=1}^{N} H_k^{\circ} \left[Y_{ko}(1 - Z_o) - Y_{k\infty} \right]$$

$$\hat{E} \equiv E - \dot{m} \sum_{k=1}^{N} Y_{k\infty} H_k^{\circ}$$

and

$$H_k \equiv h_k^{\circ} + c_p (T_{fo} - T^{\circ})$$

Combining Eqs. (5.25) and (5.21) one obtains finally:

$$\frac{dT_f}{dx} = \left(\frac{\dot{m}}{\lambda}\right) \left\{ -\frac{\hat{E}}{\dot{m}} + \frac{Z}{Z_o}\hat{Q} + (1-Z)\left[C_p(T_f - T_{fo}) - \frac{u^2}{2}\right] + Z\frac{V^2}{2} - ZuV + u\frac{P}{\dot{m}} - (1-Z)RT_f \right\} \tag{5.26}$$

Eqs. (5.13), (5.15), (5.21) and (5.26) can be used to estimate the distances, L, required for Z, V, u and T_f to change by appreciable fractions of their initial values. For a C-J detonation in a spray composed of 30 μ radius droplets in air, Williams found that $L_Z \sim 10^2$ cm, $L_V \sim 10^4$ cm, $L_u \sim 10^{-4}$ cm, and $L_T \sim 10^{-4}$. Hence there is negligible vaporization or change in droplet velocity in the region where u and T change rapidly, suggesting that the NDZ detonation structure of a shock followed by reaction zone may well apply to spray detonations.

As Williams points out, the large value of L_Z casts serious doubt on the stability of spray detonations supported only by heterogeneous

reactions, since interaction of the deflagration zone with the walls would probably be much stronger than its interaction with the shock wave. When $r < 1\mu$, L_z becomes sufficiently small to assure a significant role for the interaction between the shock and the reaction zone. With such small radius the droplets have been found to burn essentially like a vapor [31] . Droplet shattering, which can be expected for $r > 20\mu$, may play then an important role in the propagation of the wave.

This is, in fact, the conclusion reached by Webber [29] and Cramer [30] who studied heterogeneous combustion in a shock tube. Webber [29] found that a high amplitude pressure wave could be sustained in a spray of fairly coarse droplets of relatively non-volatile liquid fuel, which was attributed by Cramer [30] to the high rate of energy release resulting from burning of the shear-produced microspray in the gas flow field behind the shock.

The value of the contribution that can be derived from experiments performed in detonation tubes towards better understanding of heterogeneous combustion involving sprays has been clearly demonstrated, especially in so far as the effects of interactions with pressure waves are concerned, but the exploitation of this tehcnique has not been yet fully realized.

6. Detonation and Advanced Propulsion Systems

The significance of the background knowledge gained by our study is perhaps best illustrated by its application to advanced propulsion systems which are completely devoid of a detonative process. Basically this stems from the fact that the mechanism of the generation of pressure waves that is essential to the development of detonation plays also an important role in any device that transforms internal energy into kinetic energy in a gaseous medium. While in steady flow systems it must be considered in connection with the starting process and the occurrence of instabilities, it acquires a dominant position for any intermittent mode of operation.

With reference to nuclear propulsion it is of particular interest to note that gaseous fission reactors are currently considered for booster propulsion (Meghreblian [32]), and a number of schemes has been already proposed for the development of fusion rocket engines (Hilton, Luce and Thompson [33]). While the latter is still in such an early conceptual stage that any consideration of transient phenomena is premature, the former is already sufficiently well defined that analysis and experiments on the generation of pressure waves in a fissionable gas become quite attractive. As shown by Smith, Busch, and Oppenheim [34] there exists a marked similarity between this process and that of flame acceleration in an explosive gas mixture.

Consequently they propose to study the non-steady gasdynamic phenomena produced by neutron induced nuclear reactions in a gaseous

medium using essentially the same technique as that employed for the investigation of accelerating flames in an explosive mixutre to yield an insight into the mechanism of the generation of pressure waves, as exemplified here by Fig. 5. For this purpose the use of boron trifluoride is envisaged. It is expected that, if this substance, fully enriched with boron-10 isotope, is enclosed in a long tube, it will undergo the reaction $B^{10}(n, \alpha)Li^{7}$ as a result of a 1000 megawatt neutron pulse of approximately 10 millisecond halfwidth (an integrated flux of 2×10^{4} neutrons/cm^{2}) directed along part of its length by irradiation from a TRIGA reactor. This will produce a pressure wave which, according to the analysis of Smith, et al [34], should have an amplitude approximately three times higher than the initial pressure, with a corresponding tenfold increase in temperature, and coalesce into a shock of Mach number 1.8 at a distance of about 17 feet from the reactor core. Measurement of the pressure wave should provide information on the amount of reversible heating that can be obtained from a gaseous nuclear reaction, the reversibility in this case being defined in terms of the gasdynamic effects that are observable and analysed. Since the pressure wave can be looked upon as the source of driving force for propulsion, the experiment should be considered of particular value as one of the introductory steps in the acquisition of basic data for the development of propulsion systems based on the use of fissionable gases. The most likely choice for this purpose will be uranium hexafluoride which, unlike boron can reach criticality. It is hoped, therefore, that the hazards associated with its use will be

obviated as a result of experience gained by the proposed experiments with the much safer boron trifluoride whose fission fragments are non-radioactive.

As far as electrical propulsion systems are concerned, pulsed plasma accelerators (Gloersen [35]) are certainly pertinent to the subject matter of this paper. This is illustrated by the recent work of Rosciszewski and Oppenheim [36] who investigated the acceleration of a shock wave in a plasma as a result of its interaction with an electro-magnetic field. The shock wave in this example is driven not only, as the detonation, by energy release, which in this case is derived from the Joule heating, but also by a direct momentum gain that is provided by the Lorentz force. The plasma for this purpose was considered to have a sufficiently low conductivity so that the coupling between flow and magnetic field could be neglected. Under these conditions the superiority of using the electro-magnetic interaction process for acceleration, as contrasted to its use for the generation of electrical energy, has been specifically demonstrated proving thus, in context to intermittently operating devices, the fundamental advantage of an accelerator application for propulsion over other possible uses of the pulsed plasma principle.

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Figure Captions

1. Power density spectrum of heat release in gaseous media.

2. Detonation wave ramjet engine [5] .

3. Intermittent detonation wave engine [11] .

4. Rotating detonation wave engine [12] .

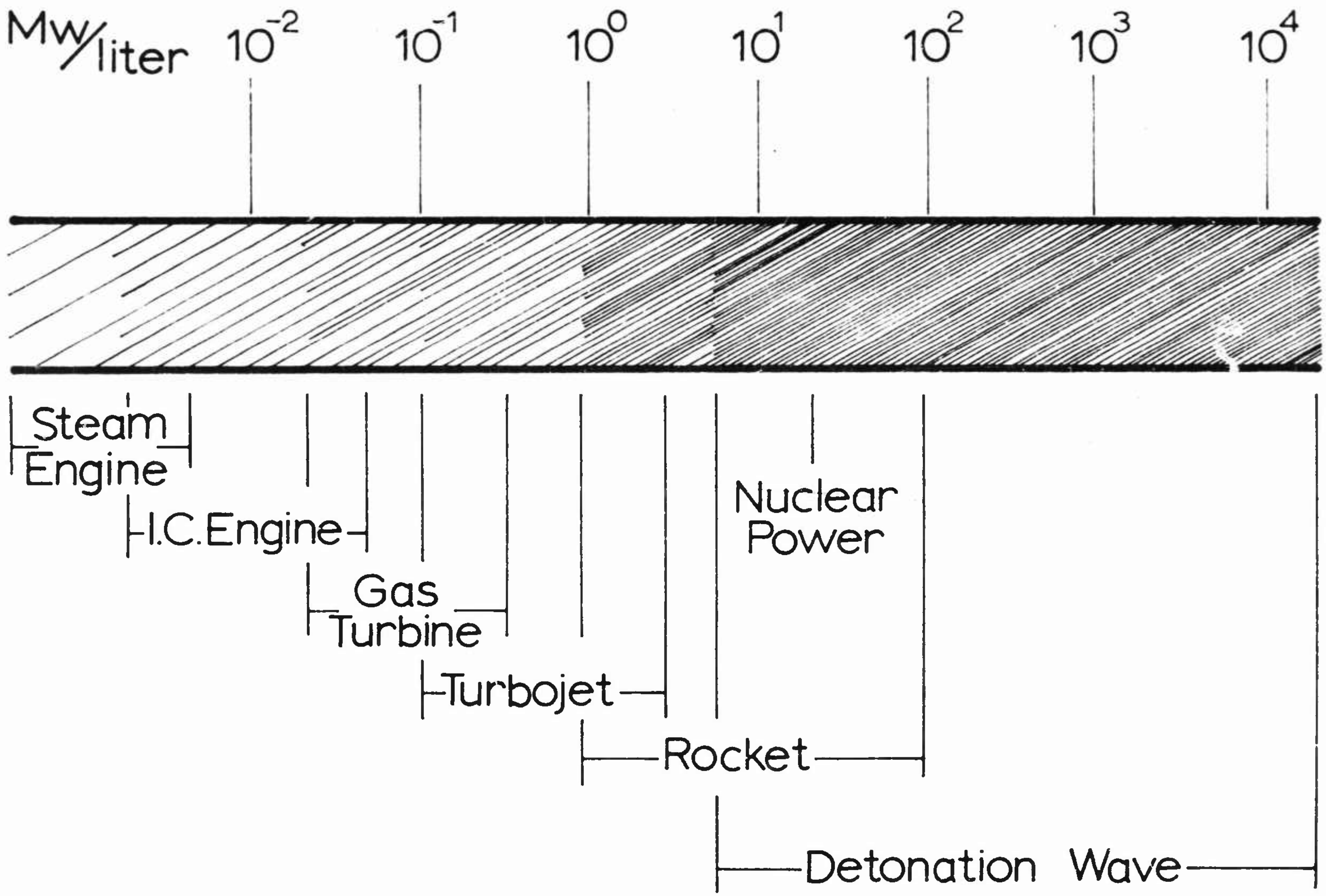
5. Streak Schlieren photograph of development of detonation in stoichiometric hydrogen-oxygen mixture initially at N.T.P. with ignition by pilot flame. Pressure records at positions 1, 2, and 3 shown as inserts. Vertical grid: 10.4 psi/div for insert 1 , 26 psi/div for inserts 2 and 3 . Horizontal grid: 50 μ sec/cm from left to right [15] .

6. Sequence of flash Schlieren photographs corresponding to the events shown in Fig. 5. [15]

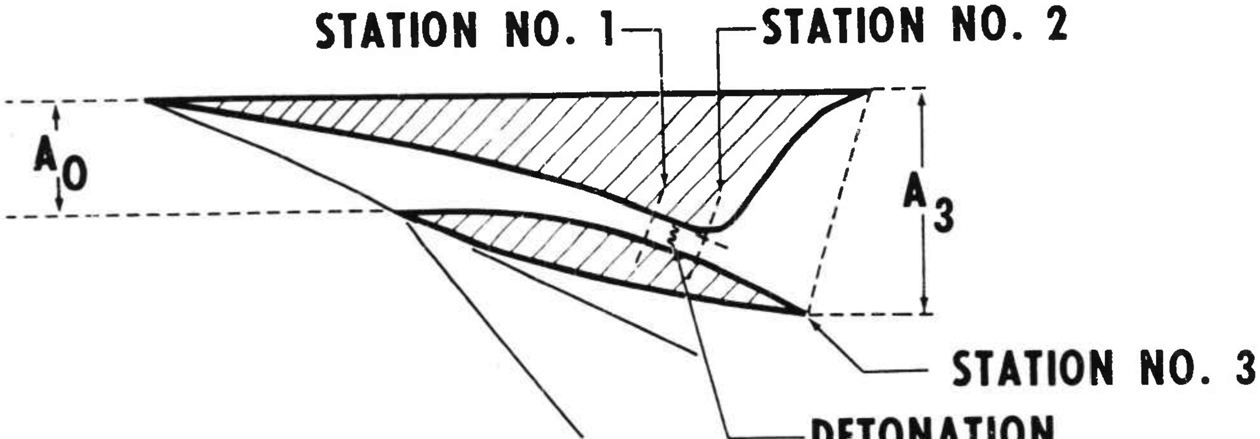
7. Interpretation of Fig. 6.

8. Streak Schlieren photograph of the onset of retonation in a stoichiometric hydrogen-oxygen mixture initially at N.T.P. with hot wire ignitor located at the closed end of tube. Abscissa scale denotes distance from ignitor. Included as inserts, and located at the time instances to which they correspond, are flash Schlieren records showing the formation of the retonation wave and the nature of the transverse waves which are generated simultaneously in the combustion zone. [16]

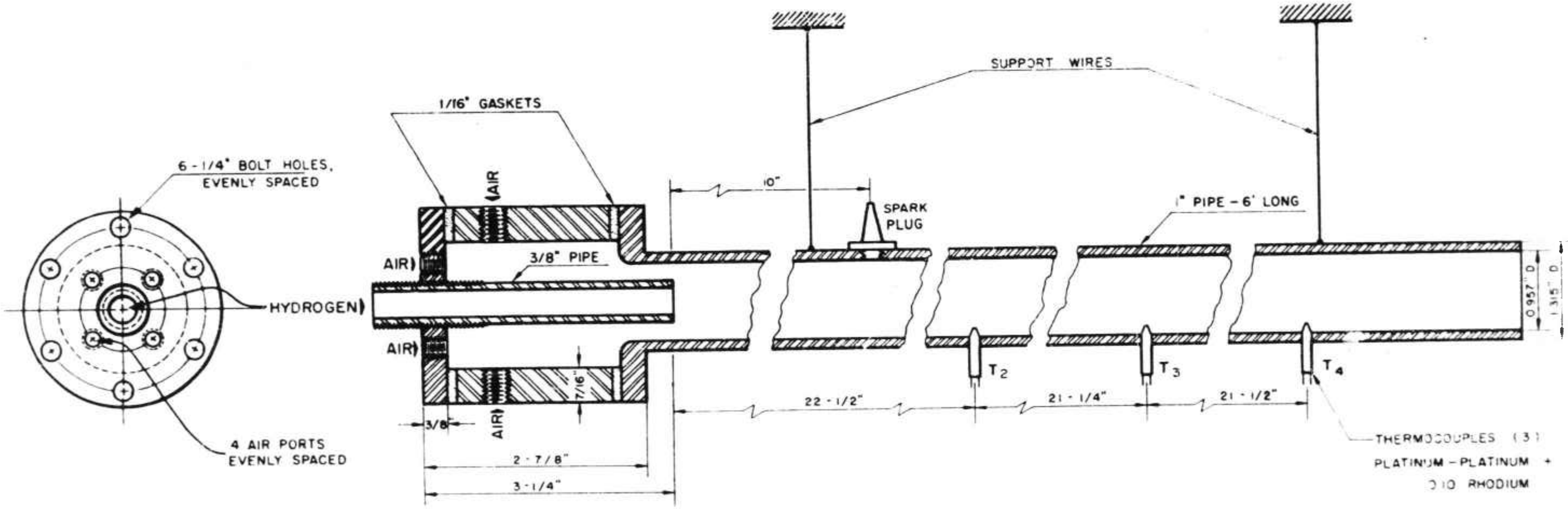
9. Examples of detonation waves with distributed reaction zones. (a) Interaction pattern produced on a soot-coated wall by detonation in a $2H_2+O_2$ mixture at an initial pressure of an order of 10cm Hg [23] ; (b) Interferogram of self-sustained detonation in a $2H_2+O_2+2CO$ mixture initially at a pressure of 0.3 atm, showing the apparently turbulent character of the combustion zone [25] ; (c) Interferogram of laminar detonation in a $2H_2+O_2+2CO$ mixture at an initial pressure of 10 mm Hg, produced by passing a self-sustained wave through a convergent-divergent nozzle [26] . In both interferograms increase in density displaces fringes upward.

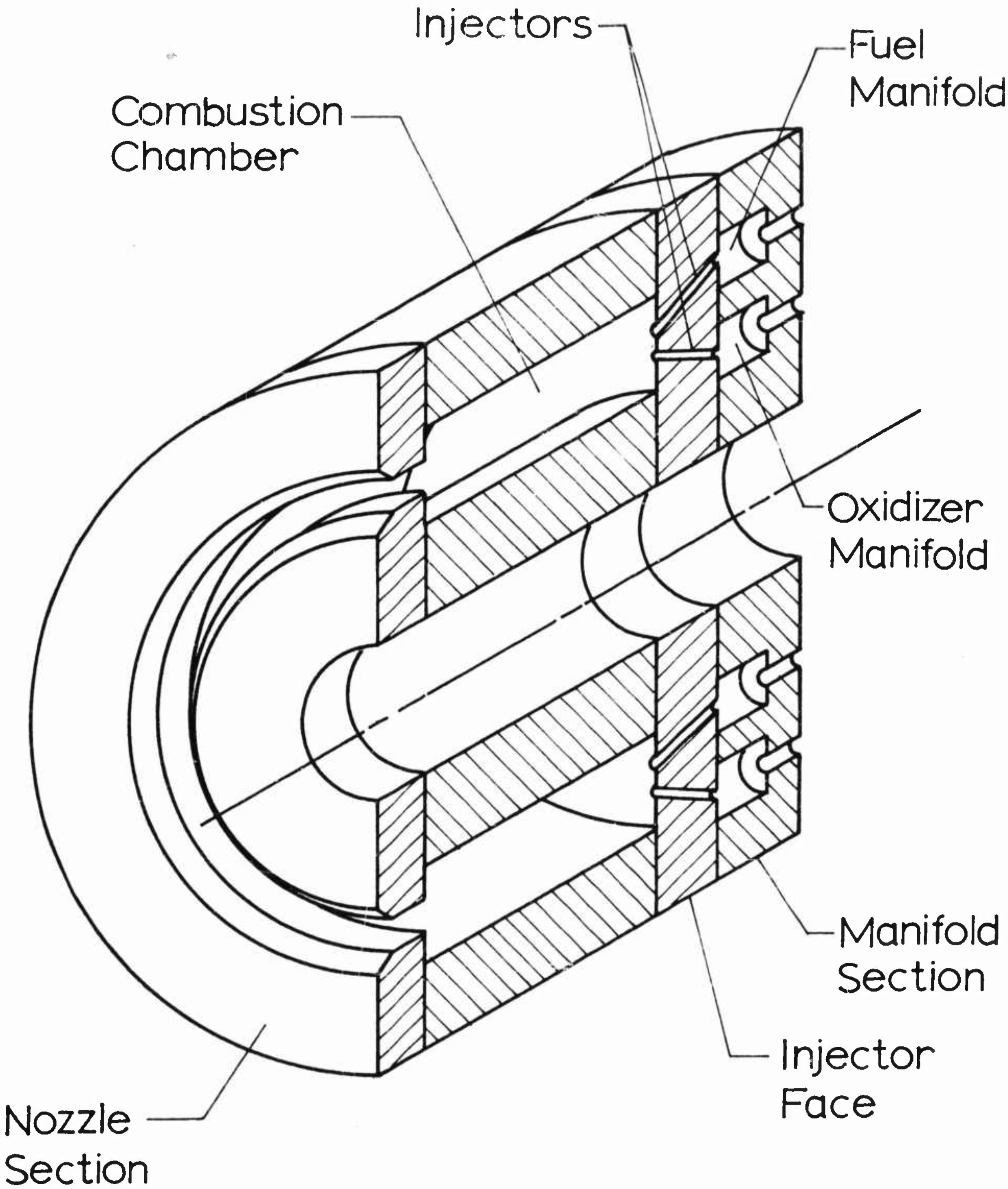


Mw/liter
10-2
10-1
100
101
102
103
104
Steam Engine
I.C. Engine
Gas Turbine
Turbojet
Nuclear Power
Rocket
Detonation Wave



Altitude 130,000 feet, ICAO atmosphere
Hydrocarbon fuel
Chapman - Jouguet conditions at detonation
Mach number ahead of detonation - 4.0
Non - dissociative case


6 - 1/4" BOLT HOLES, EVENLY SPACED
HYDROGEN
4 AIR PORTS EVENLY SPACED
1/16" GASKETS
AIR
AIR
AIR
AIR
3/8" PIPE
7/16"
3/8
2 - 7/8"
3 - 1/4"
10"
SPARK PLUG
SUPPORT WIRES
1" PIPE - 6' LONG
22 - 1/2"
21 - 1/4"
21 - 1/2"
T2
T3
T4
0.957" D
1.315" D
THERMOCOUPLES (3)
PLATINUM - PLATINUM + 0.10 RHODIUM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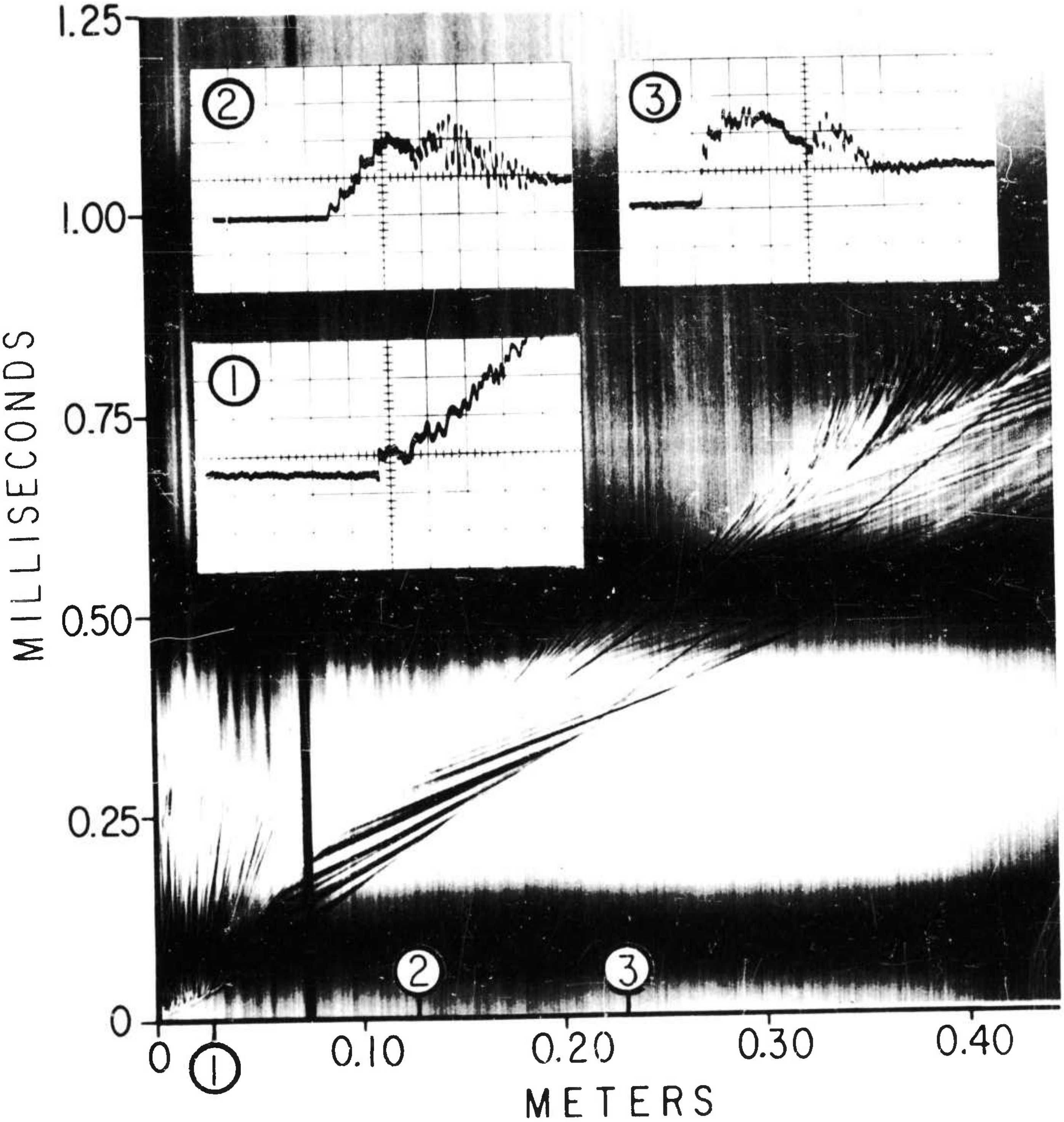
Fuel Manifold
Combustion Chamber
Oxidizer Manifold
Manifold Section
Injector Face
Nozzle Section



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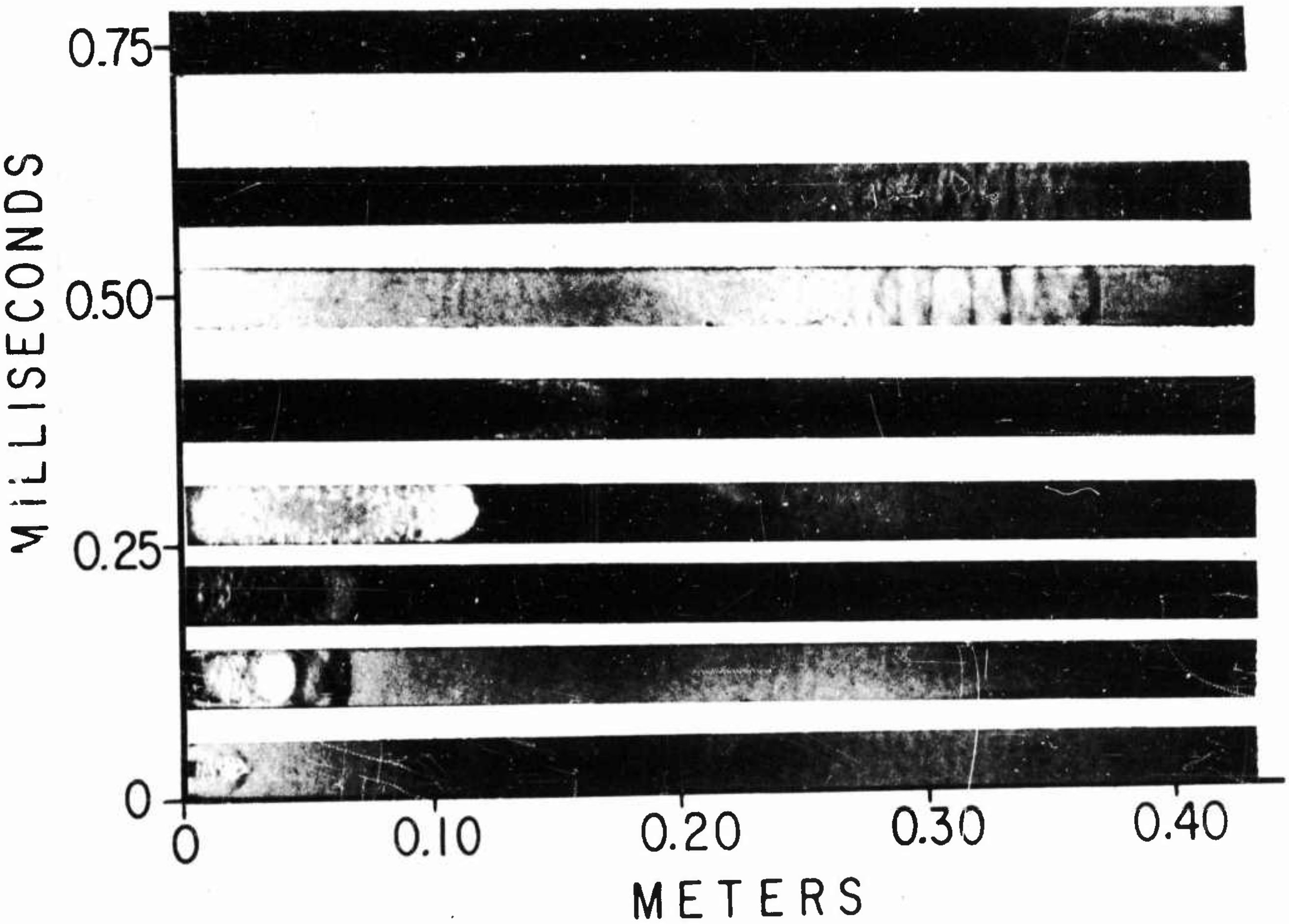
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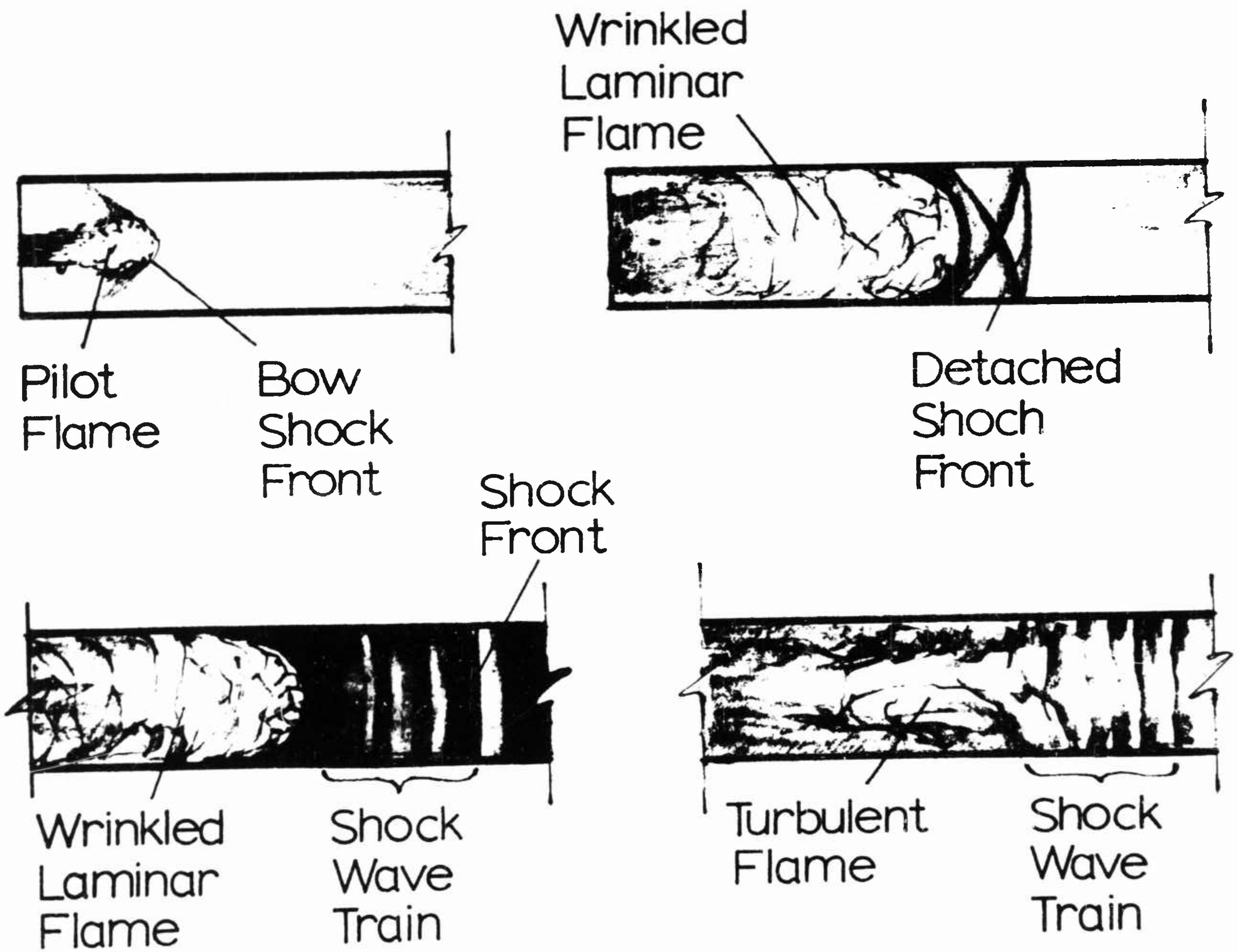
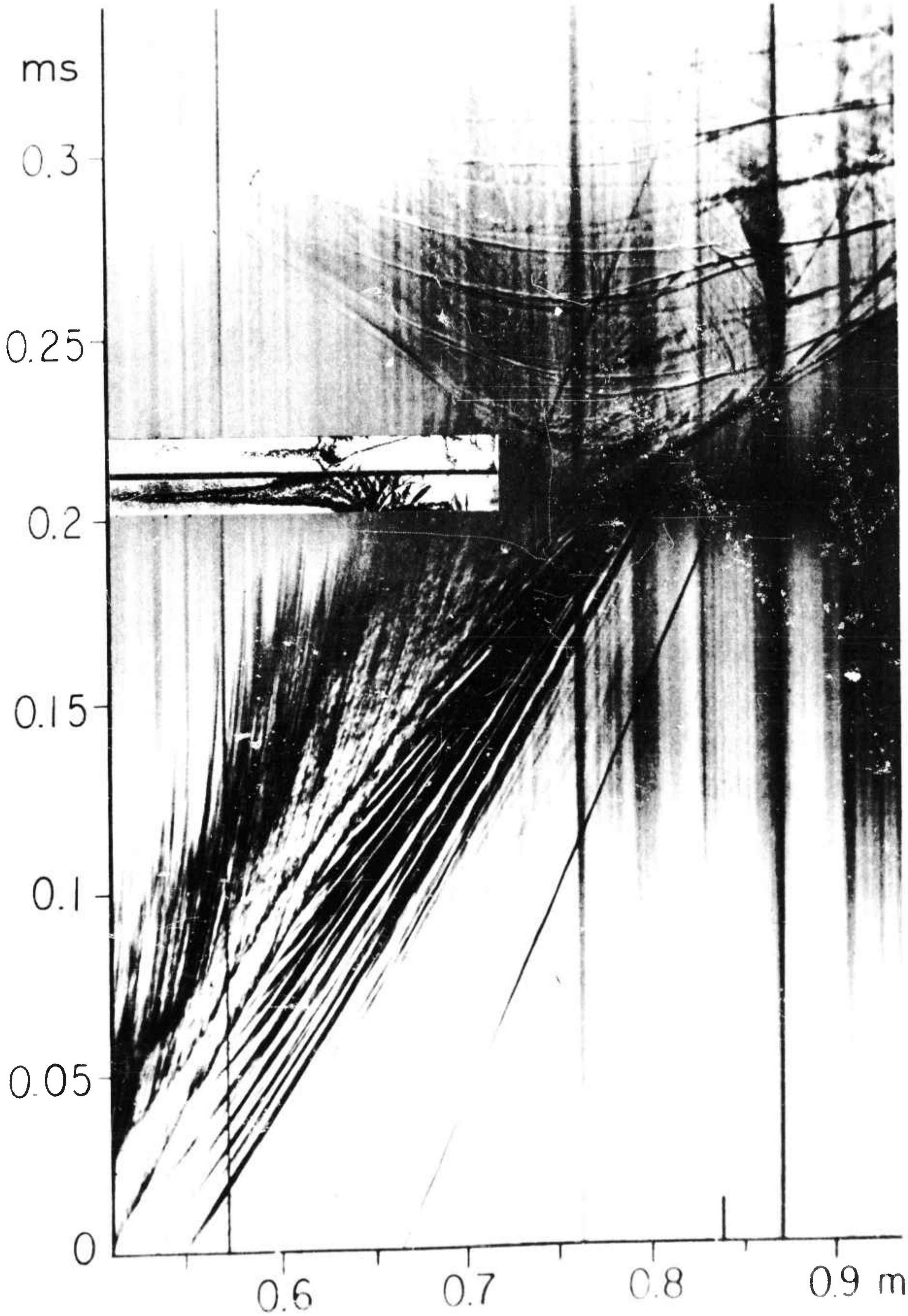


Wrinkled Laminar Flame
Pilot Flame
Bow Shock Front
Detached Shoch Front
Shock Front
Wrinkled Laminar Flame
Shock Wave Train
Turbulent Flame
Shock Wave Train



ms
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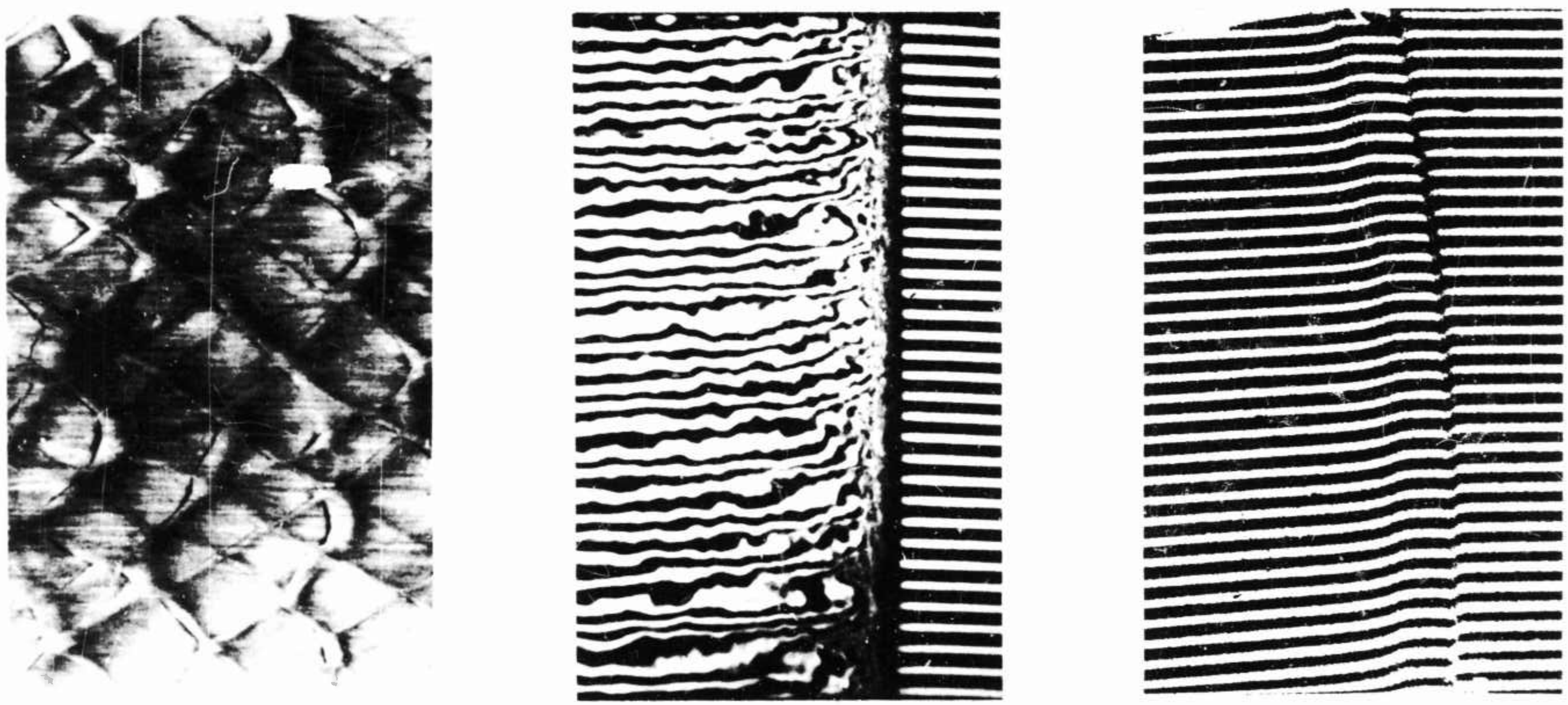

a. b. c.